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CASH MANAGEMENT POLICIES BY EVOLUTIONARY MODELS: A COMPARISON USING THE MILLER-ORR MODEL

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ABSTRACT

This work aims to apply genetic algorithms (GA) and particle swarm optimization (PSO) to managing cash balance, comparing performance results between computational models and the Miller-Orr model. Thus, the paper proposes the application of computational evolutionary models to minimize the total cost of cash balance maintenance, obtaining the parameters for a cash management policy, using assumptions presented in the literature, considering the cost of maintenance and opportunity for cost of cash. For such, we developed computational experiments from cash flows simulated to implement the algorithms. For a control purpose, an algorithm has been developed that uses the Miller-Orr model defining the lower bound parameter, which is not obtained by the original model. The results indicate that evolutionary algorithms present better results than the Miller-Orr model, with prevalence for PSO algorithm in results.

Keywords: Cash Flow, Cash Balance, Treasury, Genetic Algorithms, Particle Swarm Optimization.

1. INTRODUCTION

The management of the cash available is a constant problem in all types of organizations. This is because of daily cash inflows and outflows, either by operating activities of the company or financial transactions that have been negotiated. So, there is a need to control financial resources in order to obtain the best result for the firm.

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Thus, the function of cash management has the responsibility to mobilize, control and plan the financial resources of companies (Srinivasan & Kim, 1985). The use of models to support decision making becomes relevant, since they can provide a comprehensive view and optimization, which can hardly be obtained without the use of methodologies for this objective.

The use of models in the problem of defining the optimal level of available cash had its origin in the work of Baumol (1952) and Tobin (1956), where the authors start from the assumption that cash balance available may be defined as a commodity in inventory, i.e. a standard good, whose control may be daily, weekly, monthly, etc. depending on the level of temporal detail required by the company.

For these authors, the definition of the optimal cash balance follows the form of models to control inventory size, where it is considered the financial resource available as an inventory that has certain costs associated with its origin and maintenance, but also generates benefits indispensable to the firm.

The definition of cash balance began to have a quantitative approach in order to promote the optimization of this financial inventory in order to minimize the costs associated with the maintenance or absence of cash available. Later, Miller and Orr (1966) defined cash balance as having an irregular fluctuation, being characterized as a random variable and they propose a stochastic model to manage the cash balance.

1.1 Objectives

The study aims to present a comparison between two computational methodologies for determining the policy of cash management, taking as a basis the structure of the model proposed by Miller and Orr.

The objective of this research is to develop a management policy of cash balance cash, based on the assumptions of cost minimization by applying genetic algorithms (GA) and particle swarm optimization (PSO) and comparing the results with the traditional Miller-Orr model.

To achieve the proposed objective the following quantitative methodology is used:

- Simulate time series of cash flows, based on assumptions noted in the literature on this topic;
- Develop computational algorithms based on genetic algorithm and particle swarm optimization which have as an objective function the minimization of maintenance costs (opportunity cost), cash balance and the cost to transfer the case to an alternative investment with high liquidity, as well as the rescue from this investment to cash;
- Perform experiments with the algorithms developed in the cash flows and comparatively analyze their results, observing advantages and perspectives. Moreover, an optimization algorithm that tests all possibilities of minimum cash will serve as a basis for checking the quality level of the models relative to the Miller-Orr model.

1.2 Relevance

Understanding the reasons that lead firms to have the need to maintain cash resources is critical to better financial management. Accordingly, Brealey and Myers (2005) suggest four reasons for the maintenance of cash balance:

1. Transactions – funds held in cash to fulfill commitments because of the temporal mismatch between the outputs (payments) and inflows (receipts) of money;
2. Precautionary – funds held in cash as maintaining a safety reserve for contingencies;
3. Speculation – funds held in cash to take advantage of opportunities to obtain discounts or favorable applications; and
4. Bank reciprocity – funds held in current accounts to meet the requirements of some banks as compensation.

The factors that lead the organization's management to take a decision on the definition of the amount of money to be kept in cash is not so easily understood or performed, as it depends on economic factors such as availability of access to resources in financial markets (credit market or capital market), cost of capital and time involved in negotiating access to resources (Opler et al., 1999), which are the main limiting factors occurring in the cash management.

In the Brazilian case, Economática data for the period 2004-2008 indicate that Brazilian firms (non-financial activity) with publicly traded shares obtained a weighted average balance of cash of 8.85% over the period (Table 1).

Brazilian Firms	**2008**	**2007**	**2006**	**2005**	**2004**
% Cash Available (Mean)	9.10%	11.39%	9.22%	7.49%	6.75%
Standard Deviation	15.81%	17.35%	16.27%	14.72%	13.87%
Number of Firms	567	369	366	350	353

Table 1 – Share of total assets in cash - Brazilian companies (elaborated by the authors, Source: Economática).

1.3 Research Problem

Taking into consideration the aspects previously reported, as well as the importance of managing the cash balance, this paper describes and analyzes the following question: Which is the best method between the traditional Miller-Orr model, or the evolutionary genetic algorithm and particle swarm optimization models, to define a policy for managing cash balance, considering the costs involved in maintaining and obtaining cash?

As this paper focuses on the qualitative methodology of financial management, so we used the techniques of genetic algorithm and particle swarm optimization in the development of the cash management policies, requiring to introduce the concepts applied to the problem dealt with and the proposed methodology for its resolution.

2 Literature Review

Presented below are the theories that provide support for this work, first reviewing the concepts of management the cash balance and further the models of genetic algorithms and particle swarm optimization.

2.1 Models for Cash Management

Cash management models had their origin in the work of Baumol (1952), the author draws a parallel between cash and other business inventories, using an adaptation of the model of inventory management known as economic order quantity (EOQ), which aims to find the best trade-off between advantages and disadvantages of owning inventories.

Nevertheless, the EOQ has restrictions when using the assumptions of fixed and predictable demand, as well as instant supplies when applying for replacement inventory (Slack et al., 1997).

According to Baumol (1952) cash inventory can be seen as an inventory of a way of trade. In the EOQ model adapted to optimize cash the optimal configuration is achieved according to the relationship between the cost opportunity and the transfer cost. In the transfer model costs increase when the company needs to sell bonds to have more cash, as the opportunity costs increase with the existence of the cash balance, it is an application that has no profitability (Ross, Westerfield & Jaffe, 2002).

The model makes the analysis of the cost associated with maintaining cash, i.e., the opportunity cost determined by the interest rate that the company no longer receives by not applying the resources, and the cost of obtaining the money for the conversion of investments into cash (Ross, Westerfield & Jaffe, 2002). The transfer cost represents expenditure incurred in application or redemption of funds, such as fees and taxes.

Later, Miller and Orr (1966) present a model that meets the randomness of cash flows, while still considering the existence of only two assets, cash and investment, and the latter is an option of low risk and high liquidity (Figure 1).

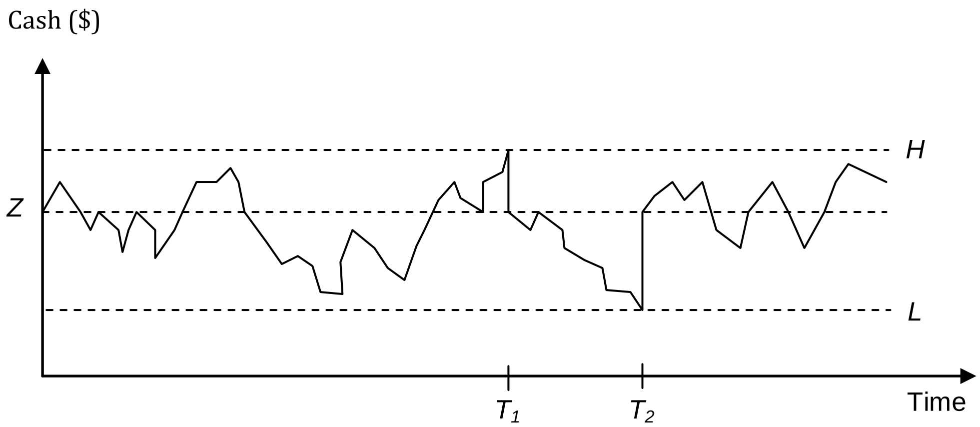

Figure 1 – Variation of cash flows, adapted (Miller & Orr, 1966).

This model seeks to define two bounds for the level of cash resources: the minimum and maximum, so when you reach the upper bound (moment T_1), represented by the high limit (*H*), investing an amount of the money in that provides the cash balance back to the optimal level of cash (*Z*). And to reach the minimum limit (moment T_2) in lower bound (*L*) should be made a rescue of cash to obtain the optimal level again (Ross, Westerfield & Jaffe, 2002).

Thus, by working the net cash flows (inputs minus outputs) the Miller-Orr model enables the cash optimization, based on the transfer costs (represented by *F*) and opportunity (represented by *K*), obtaining the following formulation (Ross, Westerfield & Jaffe, 2002):

$$Z^* = \sqrt[3]{3F\sigma^2 / 4K} + L$$

The "*"denotes optimal values and σ^2 is the variance of net cash flows. Even with the gain in relation to the Baumol model, considering randomization of cash flows, the Miller-Orr model assumes the definition of the lower bound (L), i.e. the risk of lack of cash, associated with a minimum margin safety depends on a management decision and is not treated in the model.

At this point the problem addressed in this work lies, since the Miller-Orr model itself does not define the lower bound, it is the use of optimization algorithm in this problem setting the lower limit of optimal (L^*), testing all possible L, with two decimal, to be able to minimize the cost.

Later, most of the work done uses the same assumptions as in the original models, particularly the Miller-Orr, differentiating by a stochastic modeling of the problem, as the research developed by Tapiero and Zuckerman (1980), Milbourne (1983), Hinderer and Waldman (2001), Baccarin (2002), Premachandra (2004), Volosov et al (2005), Liu and Xin (2008) and Baccarin (2009).

Few works use a computational method for solving the problem, as proposed by Yao, Chen and Lu (2006) that addresses the fuzzy systems as well as Gormley and Meade (2007) on the use of genetic algorithms, not being observed in the literature the application of PSO in this kind of problem.

2.2 Genetic Algorithms and Particle Swarm Optimization

The evolutionary computation has its origins in the study of the theory of natural evolution, models and algorithms that seek to achieve the objective functions defined for it, starting from random resolution possibilities and, according to its development algorithm, and evolving in order to obtain better results in search to the established objective (Rezende, 2005).

The algorithms of finding appropriate solutions, or optimization algorithms, use a series of assumptions or hypotheses about how to evaluate the fitness of a solution, so most of these models, based on gradient descent, depend on the occurrence of low oscillation problems or they will fail and obtain a local and non-global optimization (Moraes & Nagano, 2011).

But evolutionary algorithms do not rely on this kind of premise. Fundamentally, performance measurement should be able to order only two comparative solutions and determine the one that somehow is better than the other (Foley, 2000).

Genetic algorithms (Figure 2) population is a set of possible solutions to the given problem, each individual of this population with a similar structure to chromosomes.

GA Algorithm

```
T = O;
Generate initial population P(0);
for all each individual i o factual population P(t) do
    Evaluate fitness of individual i;
end for
while stopping criterion is not satisfied do
    t = t + 1;
    Select population P(t) from P(t-1);
    Apply cross operator on P(t);
    Apply mutation operator on P(t);
    Evaluate P(t);
end while
```

Figure 2 – General diagram of the life cycle of genetic algorithm (Rezende, 2005).

The chance of survival of each individual is evaluated by a cost function; the function to be optimized, the result of this function is the fitness of each individual as the best result to the problem, working in a selection to reproduce. Finally, evolution is provided by the application of genetic operators such as selection, crossover and mutation (Martínez et al., 2009).

The selection operators seek to determine the fitness of each individual, with the aim of obtaining the best solution to the problem; after that, individuals are crossed, i.e. by joining portions of each of fit individuals, a new population of individuals is made and eventually some individuals suffer random changes mutation, according to a given probability of occurrence (Moraes & Nagano, 2011).

The model of particle swarm optimization is more recent, and differs from genetic algorithms due to the fact that each possible solution (particle) has a random speed, drifting through hyperspace, thus each particle of the swarm is evaluated by a fitness function, with the best particle solution being stored, called *pbest*, also stored the best overall solution, *gbest* (Eberhart & Kennedy, 1995).

These features enable the PSO convergence model to the optimal result in smaller computational times. Thus, from the current position of the particle (x_i) that corresponds to the current solution, its current speed (v_i), its best past position ($pbest_i$) and the best global position of all particles in the swarm (*gbest*), each particle is updated interactively (Figure 3) in accordance with the previous attributes (Tsai et al., 2010).

PSO Algorithm (Particle Swarm Optimization)

```
Procedure – objective function (f)
Initialize the swarm of m particles
while stopping criterion is not satisfied do
    Evaluate each particle
    for particle i, i = 1, 2, . . . , m do (update the best positions)
        if f(x_i) < f (pbest_i) so
            pbest_i = x_i
            if f(pbest_i) < f (gbest) so
                gbest = pbest_i
            end if
        end if
    end for
    for particle i, i = 1, 2, . . . , m do (generate the next generation)
        v_i(t + 1) = ω v_i(t) + c_1 r_1(pbest_i − x_i) + c_2 r_2(gbest − x_i)
        x_i (t + 1) = x_i (t) + v_i (t + 1)
    end for
end while
end procedure
```

Figure 3 – Particle swarm optimization algorithm (Adapted Chen & Jiang, 2010).

There are implications for the outcome of the models according to parameters and techniques of these operators in the case of GA function selection, ordering the fittest individuals, ensuring that the best alternatives found to the problem is always maintained, since the PSO function inertia, which keeps the solution in its original path, as well as the social and cognitive behavior, seeking forward towards the solution of best results already obtained, allowing its evolution and convergence in search of the optimal result.

3 Methodology

The methodology of this work is focused on computational experiment of developing GA and PSO algorithms that are able to get the definition of the three parameters of a cash balance policy: the optimal level of cash (Z), the upper bound (H) and lower bound (L).

Therefore, it is necessary to develop experiments in different scenarios for obtaining series of net cash flows to enable the validation of the developed models.

In the specific case of the problem addressed, the referenced benchmarks in Baumol (1952), Tobin (1956), Miller & Orr (1966), Srinivasan & Kim (1986), Hinderer & Waldmann (2001), Gormley & Meade (2007) and Martínez et al. (2009) highlight the cash balance as a random variable with normal distribution.

For the experiment we used parameters of mean and standard deviation of the samples at three different levels (low, intermediate and high). The definition of the intervals follows the assumption that the two parameters that compose the normal distribution (mean and standard deviation) should vary in ranges, enabling an assessment of the sensitivity of the models to potential real effects in organizations, in order to compare them together.

The definition of these parameters, shown in Table 2, was performed empirically by previous tests, since no information supporting its definition in the literature has been found.

Thus, a total of 9 classes of problems, and for each class of problem 100 samples were randomly generated, called "Problems" with 500 value points each (Table 2). Subsequently, all the problems (900 samples for each of the 500 values) were tested for normal distribution using the Chi-Square Test (X^2) and Kolmogorov-Smirnoff Test (KS), with a significance level of 95%, while those not complying with the precept of normality were replaced before the trial.

Random Number Generation	Mean	Standard Deviation
Class 1	1,000	500
Class 2	1,000	5,000
Class 3	1,000	50,000
Class 4	20,000	500
Class 5	20,000	5,000
Class 6	20,000	50,000
Class 7	100,000	500
Class 8	100,000	5,000
Class 9	100,000	50,000

Table 2 – Random number generation

The aim was to validate the algorithms according to flows with different means and variances, obtaining flows more or less risky of presenting negative values in net cash.

An optimization algorithm was applied initially using the Miller-Orr model by changing the lower bound of cash (*L*) in order to obtain the lowest cost. The variable *L* was defined empirically between \$ 0.00 and \$ 50,000.00.

Considering the variation of *L* in \$0.01, with two decimal, for a total of 5,000,000 possible values, we obtain the value of *L* that gives the lowest cost by the Miller-Orr model.

So GA and PSO models have been applied to the problems, being programmed to minimize the cost of the cash based on the definition of the parameters *Z*, *H* and *L* simultaneously.

The development of algorithms considers the following issues:

– Initial cash balance: all series of cash balances left from a starting balance of \$ 10,000.00, plus every time the value arising from the series of cash flows. The determination of a fixed initial balance does not affect the relevance of flows, it is set just after the first calculation of the cash flow;

– The transfer cost (*F*) was set at \$ 100.00 per transaction, be it investment (cash outflow for investment) when the balance reaches the upper bound, or disinvestment (output of investment to cash) when the balance reaches the minimum limit defined. The cost of transfer corresponds to the financial cost that the company incurs when making investment and divestment operations. As the Miller-Orr model only deals with the transfer cost as being a fixed amount of currency, in this case $ 100, the other models follow the same pattern. However, in practice, it is common to the composition of the cost of transfer to be formed by a fixed amount and a percentage value on the transaction value of investment / disinvestment. The $100 value was assigned empirically based on previous tests, similar to that used by Miller and Orr in their model;

- The opportunity cost (*K*), given by the financial cost of obtaining cash when cash rupture occurs, having borrowed from the organization the obtained feature of 0.0261158% per day on this value, a rate which is equivalent to 10% per year. The opportunity cost is the interest rate that the organization would have to pay for borrowing money to make the necessary payments. The value of 10% per year was defined empirically similarly to that applied to long-term bonds and validated in previous tests;
- The values of the ideal cash balance (*Z*), upper bound (*H*) and lower bound (*L*) to be defined by the algorithms GA and PSO, should be in a sample space between $ 0 and $ 300,000, set empirically, based on the results obtained by the trial of the optimization algorithm in the Miller-Orr model;
- 100 individuals of response value were generated (*Z*, *H* and *L*) in each experiment, with 500 iterations for each (GA and PSO) to obtain the cost for each time and total cost of cash flows;
- After 10 interactions (1,000 solutions for *Z*, *H* and *L*) without the cost reduction algorithm was terminated, also considering the time as an efficiency factor.

For the GA the following parameters have been set:

- Values: binary, transformed from the series of cash balance, using 48 bits (equivalent to 6 digits);
- Crossing: Roulette method between two parents generating two children, with 70% chance of occurrence;
- Mutation: mutation rate of 1%, changing a random bit.

For the PSO the following parameters have been set:

- Values: nominal cash flows generated;
- Inertia Rate: 10%;
- Learning Rate: local optimum (cognitive behavior) and global optimum (social behavior) by 20% each.

To enable a better result in setting parameter *L*, as well as GA and PSO which use random components in each question, the experiment was performed 10 times and maintained for comparative purposes with the best result. Thus, experiments were performed with 9,000 trials with GA and 9000 trials with PSO.

The parameters used in this methodology were assigned empirically, aiming at the observation of the composition of the result of the values of *Z*, *H* and *L*, because there were no references to base the form of structure of this problem.

For comparative purposes an optimization algorithm was developed, which only changes the value of parameter *L* from $0.00 to $ 50,000.00 in value to two decimal, calculating for each *L* the total cost in each of the 900 problems according to the Miller-Orr model. This algorithm is not feasible in more complex models (with more than one variable, as in this case L), because the computational time of a combinatorial problem would make it prohibitively expensive to solve real problems.

The algorithms were developed in MATLAB® 2009 and used on a computer with Core2Quad Q8300 with 2,5GHz and 4GB of RAM memory, using Windows 7™ 64 Bits.

The following results are presented and analyzed.

4 RESULTS

The results obtained with the function of minimizing the total cost of ownership of cash, based on the lower bound (*L*) from the Miller-Orr model are presented in Optimal Algorithm.

The results using the GA and PSO algorithms, with the average values calculated over the 100 problems used in the experiments in each class problem, show the cost of cash, the iteration in which the lowest cost was obtained and the computational time per seconds by achievement.

Thus, comparative mean results of each class of problem are presented in Table 3.

Class of Problem	Optimal Algorithm Miller-Orr		GA Algorithm			PSO Algorithm		
	Cost	Time	Cost	Iteration	Time	Cost	Iteration	Time
1	4,638.63	196.04	3,932.32	18.88	2.88	3,839.03	131.44	23.54
2	5,346.83	196.25	5,294.23	4.20	3.03	4,568.75	123.24	23.46
3	20,740.28	196.51	19,170.62	48.55	2.93	18,315.77	150.22	23.34
4	18,895.27	189.63	15,079.21	13.07	2.89	14,997.96	70.78	23.24
5	19,694.68	196.09	14,992.53	8.33	2.88	14,880.98	118.48	23.52
6	24,517.72	200.66	20,166.89	13.80	2.97	19,444.35	164.92	23.60
7	36,110.19	178.35	29,800.56	3.10	2.84	29,817.45	17.06	23.28
8	37,227.33	182.92	29,813.75	13.41	2.92	29,812.04	41.68	23.36
9	40,346.49	194.90	28,990.28	13.61	3.02	28,829.36	92.06	23.91

Table 3– Comparative results between groups by Optimal Algorithm, GA and PSO

The results demonstrate that it is possible to both evolutionary computational algorithms (GA and PSO) to determine the policy of cash management with the parameters *Z*, *H* and *L* with lower costs than the Miller-Orr model optimized for variable *L*.

It is noted that the GA algorithm is almost 10 times faster than the PSO algorithm, however, both are significantly faster than the optimal algorithm applied to the Miller-Orr model, since this is a trial and error algorithm.

Comparing the cost of the cash, obtained by GA and PSO algorithms, in relation to the Miller-Orr model, we can verify an average reduction of 21.30% of the cost for the algorithm GA and 24.68% in the PSO algorithm, according to Table 4. So, the table shows the reduction of the total cost of the cash provided by computer algorithms, in monetary terms (Cost Reduction) and percentage (% Var), when compared with the cost obtained by the Miller-Orr Model.

Class of Problem	**GA Algorithm**		**PSO Algorithm**	
	Cost Reduction	**% Var**	**Cost Reduction**	**% Var**
1	706.30	18.60%	799.60	21.28%
2	52.60	1.28%	778.07	17.08%
3	1,569.66	8.24%	2,424.52	13.28%
4	3,816.06	25.33%	3,897.31	26.00%
5	4,702.15	31.38%	4,813.70	32.36%
6	4,350.83	21.65%	5,073.37	26.15%
7	6,309.63	21.17%	6,292.74	21.11%
8	7,413.58	24.87%	7,415.29	24.87%
9	11,356.21	39.18%	11,517.13	39.96%
General		**21.30%**		**24.68%**

Table 4– Comparative results of cost reduction in GA and PSO in relation to Miller-Orr model

Note that the PSO algorithm obtained a greater reduction in costs in relation to algorithm GA, mainly in classes of problems 1, 2 and 3 corresponding to the lowest mean cash flow (mean = 1,000 in these classes), indicating a higher possibility of negative cash flows.

In the case of companies with cash flows that have lower means and larger fluctuations (standard deviation), as in the case of classes 2 and 3, the algorithm GA did not make significant gains in relation to the optimized Miller-Orr model.

Thus, the average relative deviation (ARD) between each algorithm (Optimal Miller-Orr, GA and PSO) and the best solution for cash balance policy (the one with the lowest cost), provides an insight into the most efficient algorithm. Furthermore, it is considered the number of times that each algorithm has a better solution, indicating that it is more effective to 900 problems, according to Table 5.

Class of Problem	ARD Optimal Algorithm Miller-Orr	ARD GA Algorithm	ADR PSO Algorithm	Best Solution GA	Best Solution PSO
1	24.74%	5.54%	3.04%	42.00%	58.00%
2	17.09%	15.94%	0.00%	0.00%	100.00%
3	13.28%	4.68%	0.00%	0.00%	100.00%
4	26.47%	0.93%	0.38%	43.00%	57.00%
5	32.68%	0.99%	0.24%	31.00%	69.00%
6	26.15%	3.73%	0.00%	0.00%	100.00%
7	21.37%	0.16%	0.22%	55.00%	45.00%
8	25.11%	0.20%	0.19%	54.00%	46.00%
9	40.09%	0.66%	0.09%	25.00%	75.00%
General	**25.22%**	**3.65%**	**0.46%**	**27.78%**	**72.22%**

Table 5– Comparative results of ARD and Best Solution

In Table 5 it can be seen that the PSO algorithm has the lowest average mean deviation overall, losing only in Class 7. Furthermore, the PSO algorithm gets the best solution in 72.22% of the time, and in classes 2, 3 and 6 it had the best result problems in 100% of the time compared with the algorithm GA.

Later we used the *t* Test for two samples assuming equal variances in order to verify that the cash costs obtained by the algorithms are significantly different at 5% level, indicating that the costs obtained with GA and PSO algorithms have the same characteristics distribution over 99% (Table 6).

t-Test: two sample assuming equal variances	***GA***	***PSO***
Mean	18,582.27	23,060.51
Variance	86,859,400.32	151,313,595.00
Observations	900	900
Stat *t*	-8.705272018	
P(T<=t) bi-caudal	7.03163e-18	
t critical bi-caudal	1.961284203	

Table 6– Comparative results of cost reduction in GA and PSO over the Miller-Orr model

Thus, despite the best results of the PSO algorithm, the costs obtained are significantly different from the algorithm AG, at a level of 7.03163e-18. In a comparison between the PSO algorithm and the Optimal Algorithm in the Miller-Orr model, the descriptive level obtained is 0.995853067, demonstrating that the costs are not significantly different.

Thus, it is possible to observe that the Miller-Orr model can be used with a trial and error algorithm to obtain the minimum cost, but even in this situation results have higher costs than evolutionary algorithms GA and PSO.

Between the algorithms, the computational time factor was dropped from the analysis because a difference of 20 seconds more between the models would not be a limiting factor. So, between the GA and PSO models should be noted as the average relative deviation (ARD), as a measure of efficiency and percentage of gain and as a measure of effectiveness. In the two measures, PSO algorithms performed better although this cost difference is not statistically significant at 5%.

5 CONCLUSION

The genetic algorithms and particle swarm optimization have been proven to be useful tools in the application this kind of optimization problem. When assisting in the definition of parameters for managing, cash balance can find with higher impartiality the optimal values for the cash management.

The analysis shows that the PSO algorithm gets lower costs with higher efficiency (ARD) and efficacy (greater number of hits), but not significantly different from each other. Regarding the computational time, the algorithm GA showed an average time of 2.93 seconds per problem, while the PSO algorithm had an average time of 23.47 seconds.

Considering that each company would be a problem, although the computational time average PSO algorithm is much higher, a difference of 20 seconds to get the firm's cash balance policy would not be something problematic.

In practice, the two kinds of algorithms are presented as a practical solution to define a policy for the management of the cash balance, obtaining significant gains in relation to cost and time obtained by the optimized Miller-Orr model. However, given the experimental results, the PSO algorithm has higher convergence in the pursuit of lower cost, within the criteria established.

This study focuses on the comparison between the Miller and Orr model and computational algorithms GA and PSO developed with the aim of setting management policy in cash, with the variables for the ideal cash (Z), upper bound (H) and lower bound (L), but the models GA and PSO can be applied for the definition of all more complex cash policies, without the limitations of the Miller-Orr model, as:

- Consider only a fixed cost in monetary cost transfer (*F*), when in practice these costs usually have a fixed component and a variable component as a percentage of the operation amount;
- Consider the same transfer cost (*F*) in investment operations and rescue, since in practice there are different costs;
- The incidence of opportunity cost (*K*) when cash resources are left without considering obtaining profitability with the use of financial resources, which would reduce the cost of cash maintenance.

So, the results point to a promising area, but further studies and experiments are needed, since the results could not be compared with other newer models, like the ones y Hinderer & Waldmann (2001), Gormley & Meade (2007) and Baccarin (2002 e

2009), because these models have shown reductions in limitations in the Miller-Orr Model.

Nevertheless, with these diversifications, it would not be possible to apply a control algorithm as we did in this study, since computation time of the definition of three parameters simultaneously would be prohibitive, hence the great relevance of this study; we present results which demonstrate that GA and PSO algorithms can be used in more sophisticated models to the problem of cash management, signs of obtaining practical solutions acceptable.

Therefore, this study presents its contribution to the validation of GA and PSO algorithms, especially with the PSO model as reliable, quick and malleable in the development of algorithms that enable the reduction of limitations, enabling the development of policies for cash management closer to reality, which are applicable for the financial management of organizations.

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